Revised 24 Feb 2021 23:00

15 Noise in Membrane Voltage

15.1 Thermal Fluctuations

Ion flow across the membrane gives rise to a conductance G across the cell. This leads to a fluctuation in the potential, known as the Johnson noise, of size

$$\delta V = \sqrt{\frac{4k_BT\Delta f}{G}} = \sqrt{\frac{k_BT}{C}} \tag{15.1}$$

where we used

$$\Delta f = \int_0^\infty df \, \frac{1}{1+(2\pi f(C/G))^2} = \frac{G}{4C}. \tag{15.2}$$

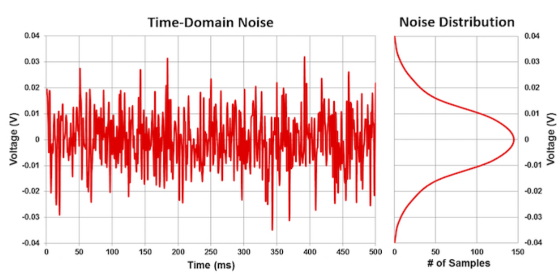
Another way to derive the equation for the thermal noise is to use the equipartition theorem to equate the fluctuating energy in the membrane to the thermal energy, i.e.,

$$\frac{1}{2}C\delta V^2 = \frac{1}{2}k_BT \tag{15.3}$$

The capacitance is given by $C = \epsilon_m$ (area/thickness), so that for a thin dielectric sphere of thickness L and radius a, $C = \epsilon_m \frac{4\pi a^2}{L}$. Thus

$$\delta V = \sqrt{\left(\frac{k_BT}{e}\right)\left(\frac{L}{\epsilon_m}\right)\frac{e}{4\pi a^2}} = \frac{1}{2a}\sqrt{\left(\frac{k_BT}{e}\right)\left(\frac{e}{c_m}\right)\frac{1}{\pi}}. \tag{15.4}$$

Figure 1: Johnson noise and Gaussian amplitude distribution



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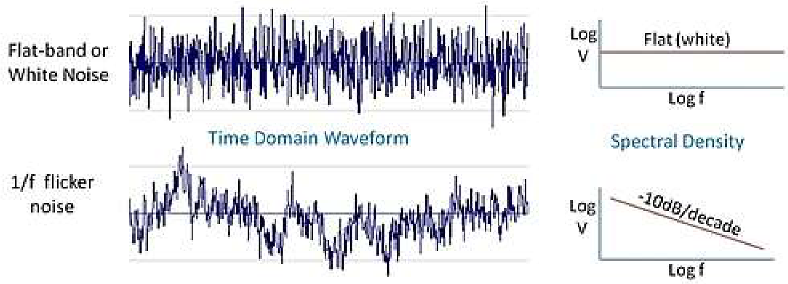
For most all cells, the ratio $\frac{\epsilon_m}{L}$ is

$$c_m \equiv \frac{\epsilon_m}{L} \approx 1.0x10^{-14}\frac{F}{\mu m^2}. \tag{15.5}$$

Figure 2: Johnson noise and 1/f noise



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For a neuron of radius $a = 20\mu m$, the noise level is found to be $\delta V \approx 20\mu V$. **The important result is that the membrane noise level for cells is much less than the thermal voltage** k_BT/e, where

$$\frac{k_BT}{e} \approx 25mV \tag{15.6}$$

Only at the smallest structure, the synaptic vesicle, or synaptosome, with outer radius $a \approx 30$ to 50 nm, is the noise level likely to approach the thermal voltage. Let's thus look at the fluctuation in the number of ions across the cell. In synaptic vesicles, the membrane potential ΔV is set by a hydrogen ion, or pH gradient. Then

$$\Delta V = \frac{k_BT}{e} ln \frac{[H^+]_{out}}{[H^+]_{in}} = \frac{k_BT}{e} \left(pH_{in} - pH_{out}\right). \tag{15.7}$$

Typically, $pH_{in} \approx 5$ and $pH_{out} \approx 7.5$. The variance in the transmembrane voltage in terms of ion concentration is

$$\delta V = \left| \frac{\partial \Delta V}{\partial [H^+]_{in}} \delta [H^+]_{in} \right| = \frac{k_BT}{e} \left| \frac{\delta [H^+]_{in}}{[H^+]_{in}} \right| \tag{15.8}$$

We equate noise level this with the expression for Johnson noise to get

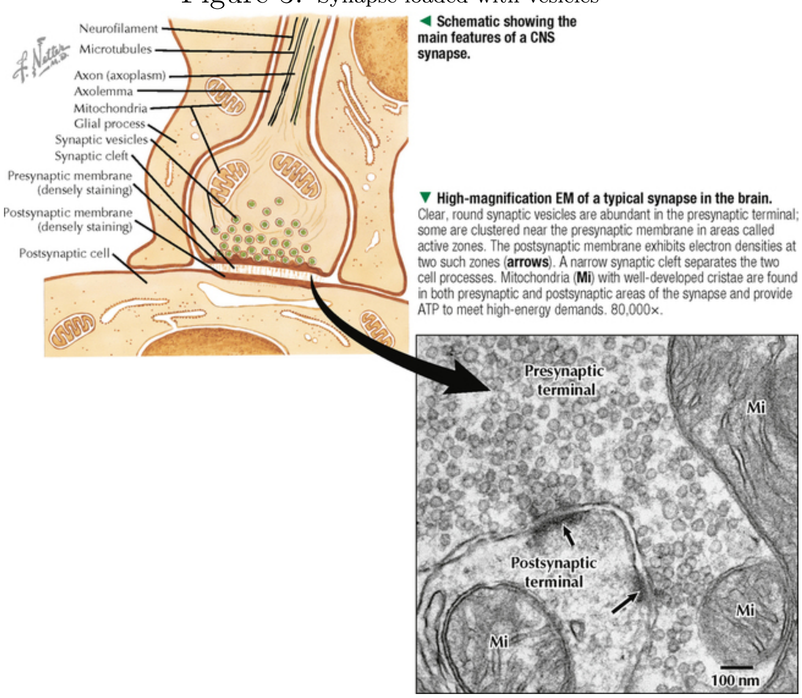
$$\frac{\delta [H^+]_{in}}{[H^+]_{in}} = \frac{e}{k_BT} \sqrt{\frac{k_BT}{C}} = \sqrt{\frac{e^2}{k_BTC}} = \sqrt{\left(\frac{e}{k_BT}\right) \frac{1}{c_m} \frac{e}{4\pi a^2}} \tag{15.9}$$

An interesting number is the value of the radius a for which the fluctuations in ion concentration are of order unity, i.e., $\frac{\delta [H^+]_{in}}{[H^+]_{in}} \approx 1$. We call this a_{crit}, where

$$a_{crit} = \sqrt{\frac{1}{4\pi} \left(\frac{e}{k_BT}\right) \frac{e}{c_m}} \approx 7nm \tag{15.10}$$

This corresponds to an inner diameter of 15 nm. The walls add about another 10 nm for a total outer diameter of $\approx$ 25 nm, which is a bit less than the observed outer diameter of vesicles. Not too bad as a limiting estimate of the smallest "cell".

Figure 3: Synapse loaded with vesicles



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